Mod4-RL1: Nonlinear Programming Primer

- Goal: introduce the basics of optimization including necessary and sufficient conditions for optimality

Unconstrained Non-Linear Programming

Suppose you are asked to find the minimum of the function $F : \mathbb{R}^m \to \mathbb{R}$; i.e., solve the nonlinear program (NLP) given by

$$\min_{u \in \mathbb{R}^m} \; F(u) \qquad \text{(NLP)}$$

The function f is called the objective.

- local minimizer: we say that $u \in \mathbb{R}^m$ is a local minimizer for NLP if there exists an open set $V \subset \mathbb{R}^m$ containing u such that $F(u) \leq F(v), \quad \forall \; v \in V$.
- strict local minimizer: we say $u \in \mathbb{R}^m$ is a strict local minimizer if the inequality above is strict
- local maximizer: we say u is a local maximizer if it is a local minimizer for

$$\min_{u \in \mathbb{R}^m} \; -F(u)$$

Sufficient and Necessary Conditions

$$\min_{u \in \mathbb{R}^m} \; F(u) \qquad \text{(NLP)}$$

Constrained Non-Linear Programming

Suppose you are asked to find the minimum of the function $F : \mathbb{R}^m \to \mathbb{R}$; i.e., solve the nonlinear program (NLP) given by

$$\begin{aligned} \min_{u \in \mathbb{R}^m} \quad & F(u) \\ \text{s.t.} \quad & f(u) = 0 \end{aligned} \qquad \text{(C-NLP)}$$

Example: Reducing to Unconstrained

Alternative: Lagrange Multipliers

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