Mod3-RL3: Observability of LTV Systems

References:

- Chapter 8.2-8.4 Callier & Desoer **[C&D]**
- Chapter 11 and 15 Hespanha **[JH]**
- **[510]** Lecture Notes (Finite Rank Operator Lemma, Hilbert Spaces, Adjoints, etc.)

Observability Map

Consider the LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t)$$

Solution is $$y(t) = \rho(t, t_0, x_0, u_{[t_0,t_1]}) = C(t)\Phi(t, t_0)x_0 + \int_{t_0}^{t} C(t)\Phi(t, \tau)B(\tau)u(\tau)\ d\tau$$

The observability map $\mathscr{L}_o : \mathbb{R}^n \to \mathscr{Y}_{[t_0,t_1]}$ is defined by

$$\mathscr{L}_o x_0 = C(\cdot)\Phi(\cdot, t_0)x_0$$

That is, $\mathscr{L}_o x_0$ is an operator in $\mathrm{PC}([t_0, t_1])$ such that

$$(\mathscr{L}_o x_0)(t) = y(t) - \int_{t_0}^{t} C(t)\Phi(t, \tau)B(\tau)u(\tau)\ d\tau$$

Observability

Definition. The state x_0 is unobservable on $[t_0, t_1]$ if and only if its zero input response is zero on $[t_0, t_1]$. i.e.,

$$x_0 \text{ is unobservable on } [t_0, t_1] \iff x_0 \in \text{Ker}(\mathscr{L}_o)$$

Theorem. Let $(A(\cdot), B(\cdot))$ be piecewise continuous. Then, we have the following equivalences:

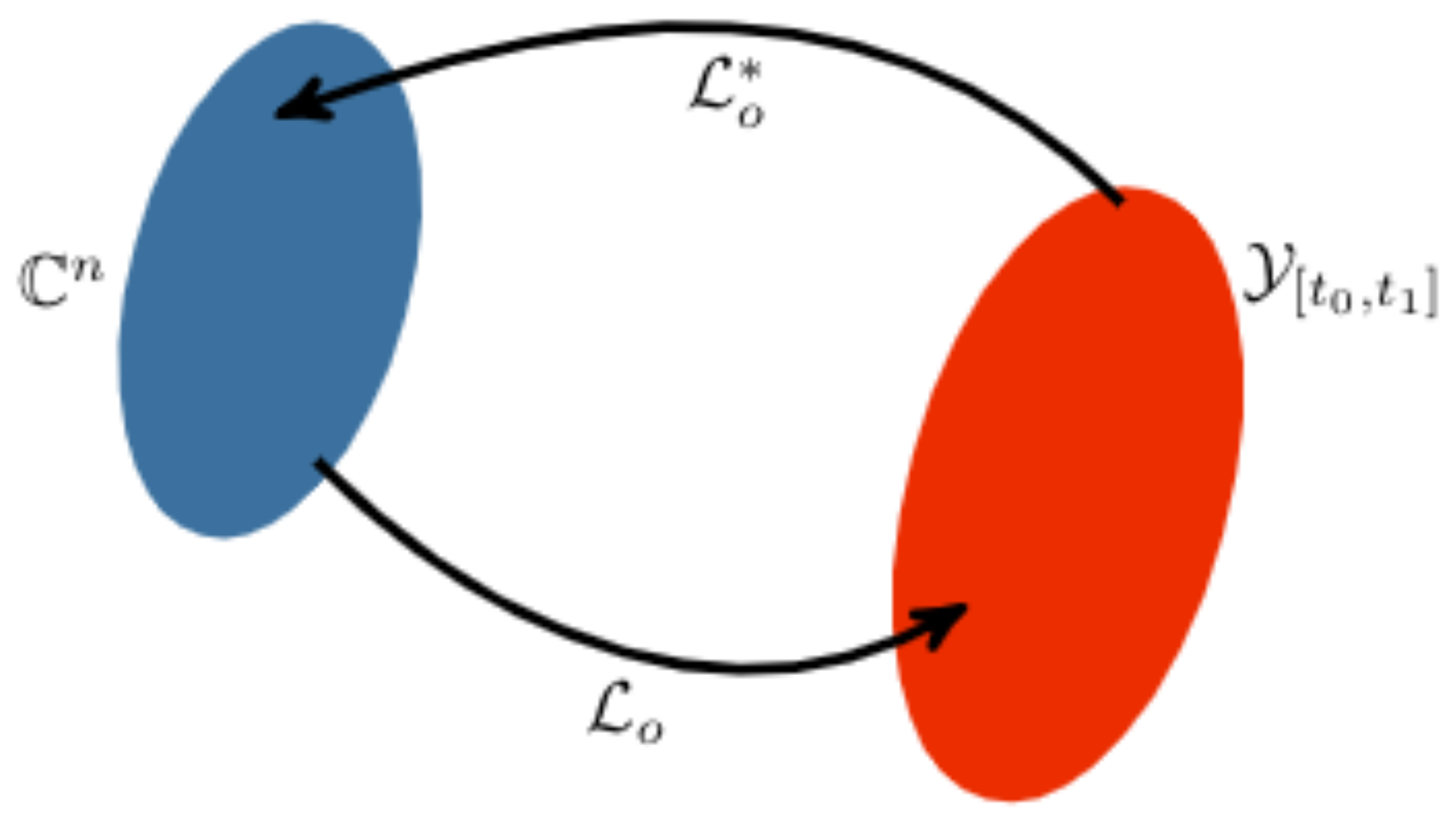
$$(A(\cdot), B(\cdot)) \text{ observable on } [t_0, t_1] \iff \text{Ker}(\mathscr{L}_o) = \{0\} \qquad (1)$$

$$\iff \text{Ker}(\mathscr{L}_o^* \mathscr{L}_o) = \{0\} \qquad (2)$$

$$\iff \det(W_{o,[t_0,t_1]}) \neq 0 \qquad (3)$$

where

$$W_{\text{o},[t_0,t_1]} = \int_{t_0}^{t_1} \Phi(\tau, t_0)^* C(\tau)^* C(\tau) \Phi(\tau, t_0) \, d\tau$$



Recovering Initial State

Example Unobservable System

Unobservable Subspace