Mod2-RL1: Introduction to Stability

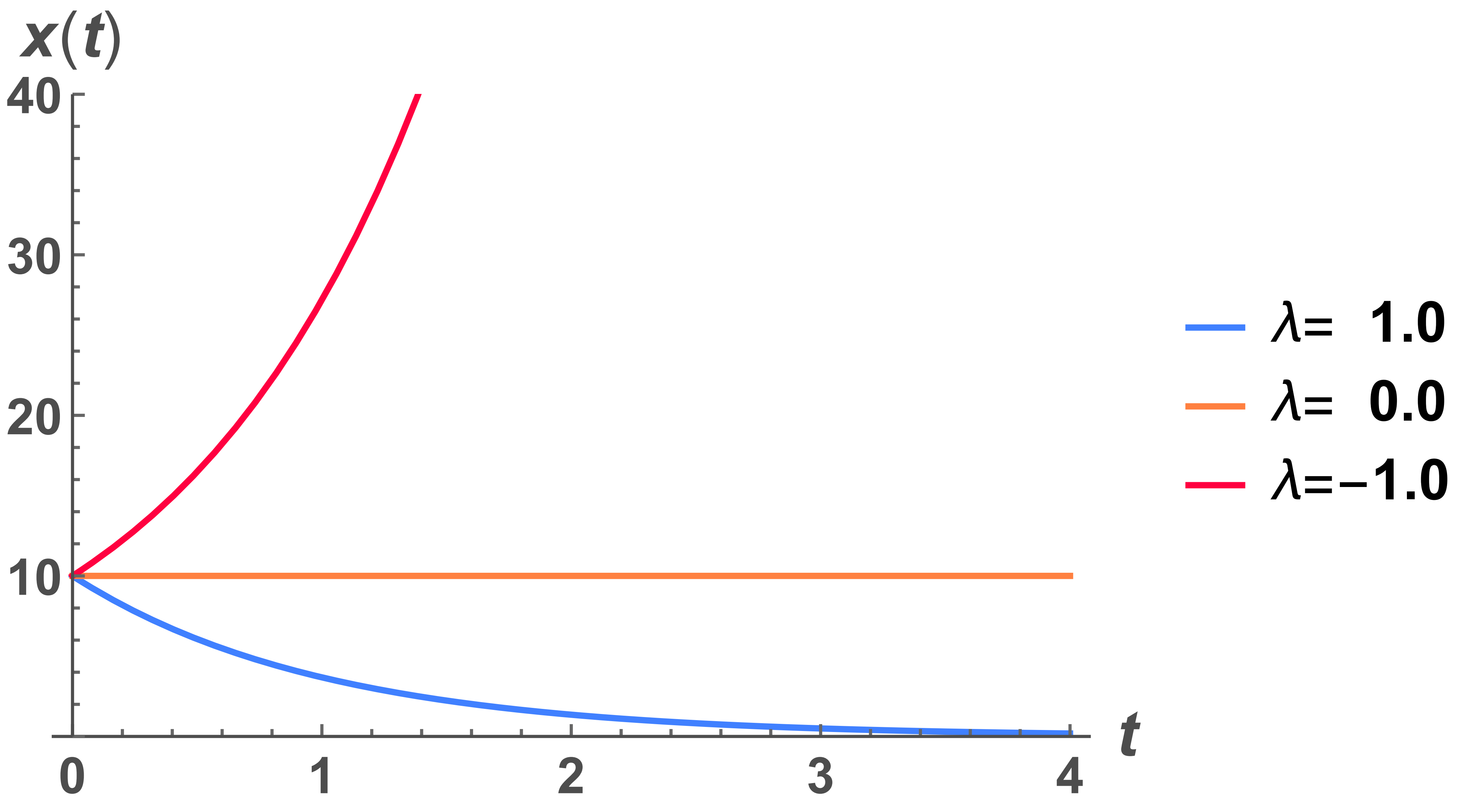
References:

- Chapter 4 and 7 Callier & Desoer **[C&D]**
- Chapter 8 and 9 Hespanha **[JH]**
- **[510]** Lecture Notes

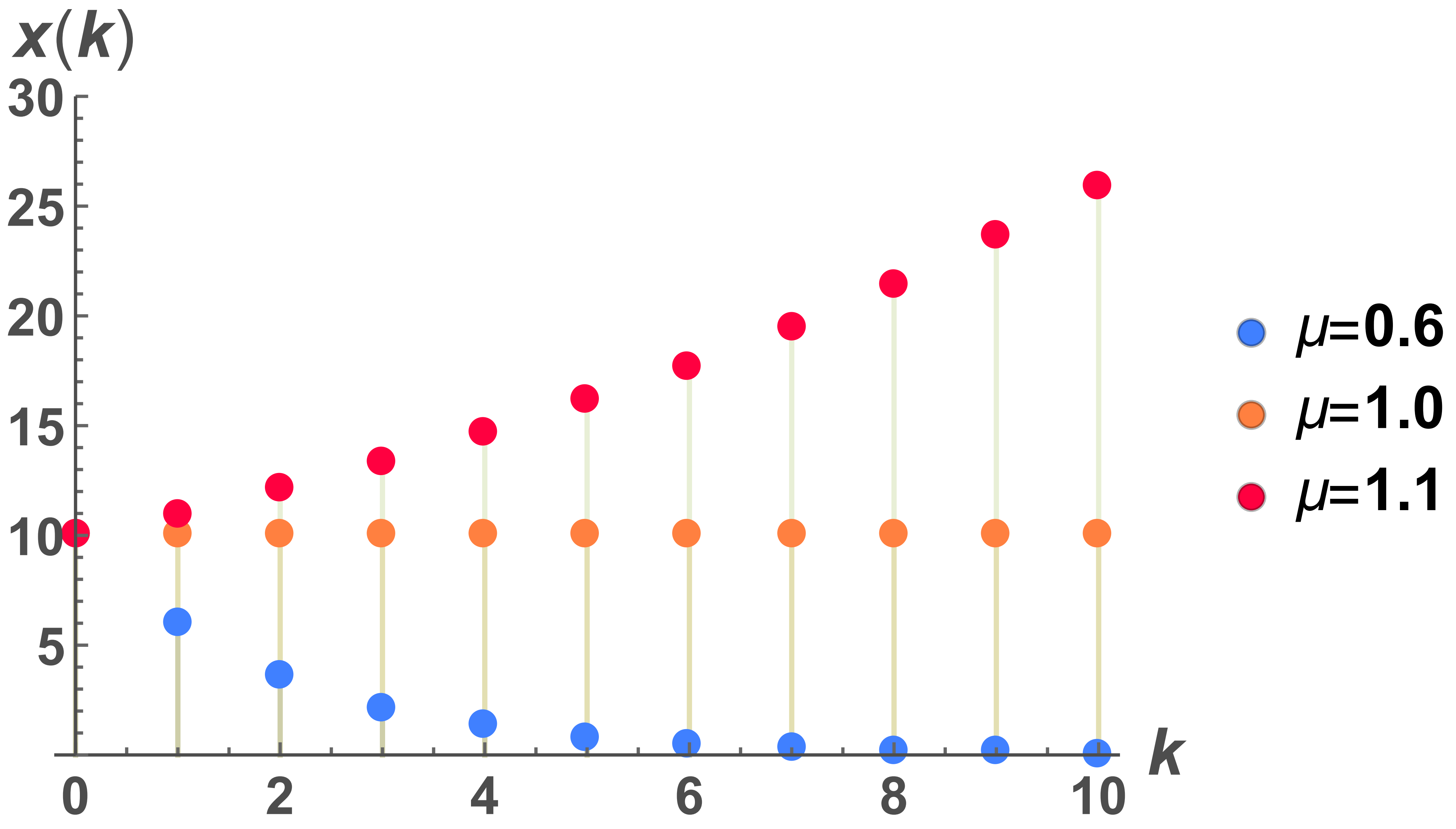
Stability: What is it?

$$\dot{x} = -\lambda x \implies x(t) = x_0 e^{-\lambda t}$$

$$x^+ = \mu x \implies x_k = \mu^k x_0$$



Continuous Time



Discrete Time

Equilibrium Point

When we talk about stability for dynamical systems $\dot{x} = f(x)$ we do so relative to the equilibrium points for the dynamics.

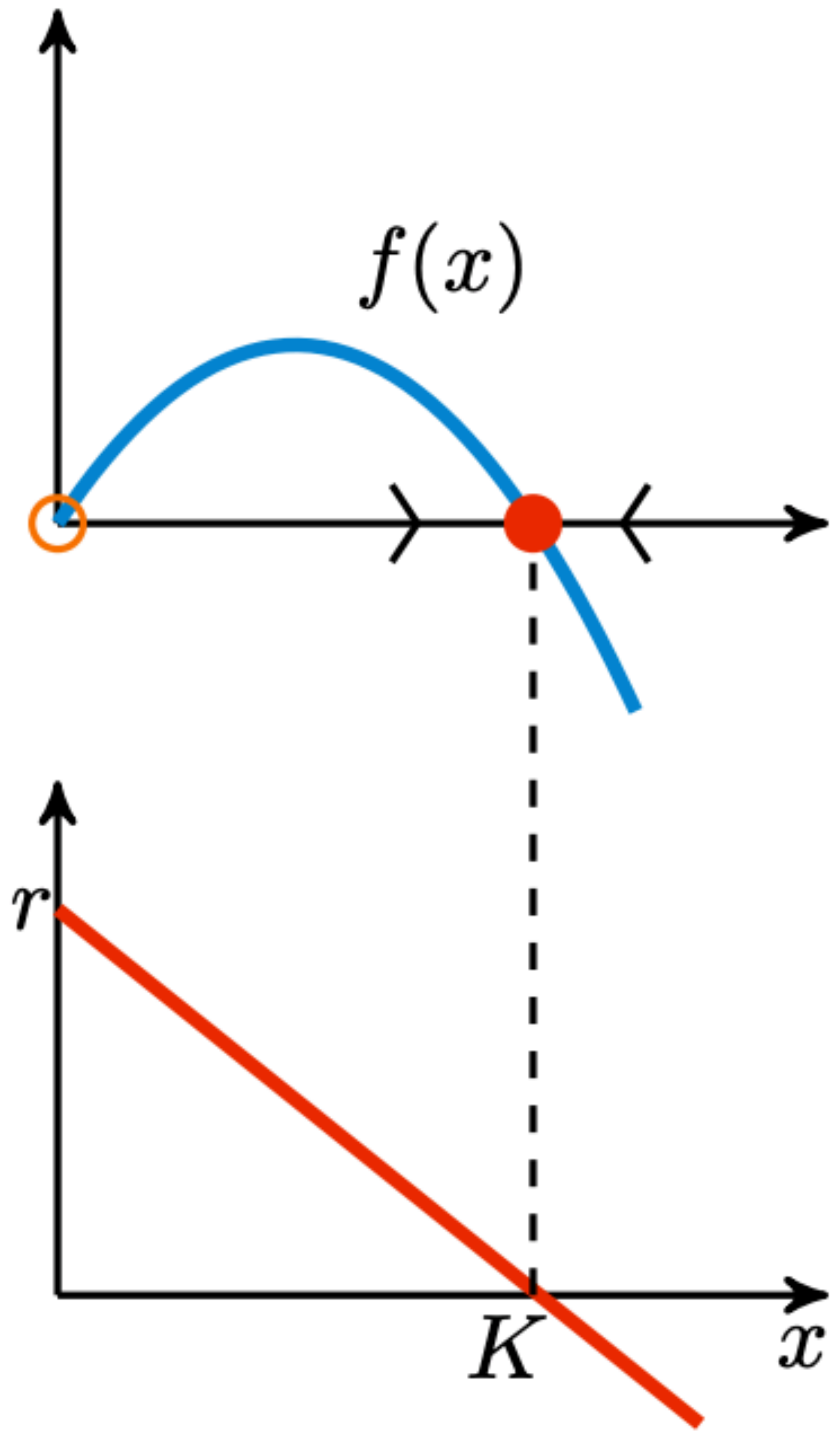
Definition: An equilibrium point x^* is a value of the state variable wherein the variables do not change given the dynamics – i.e.,

$$\dot{x} = f(x^*) = 0$$

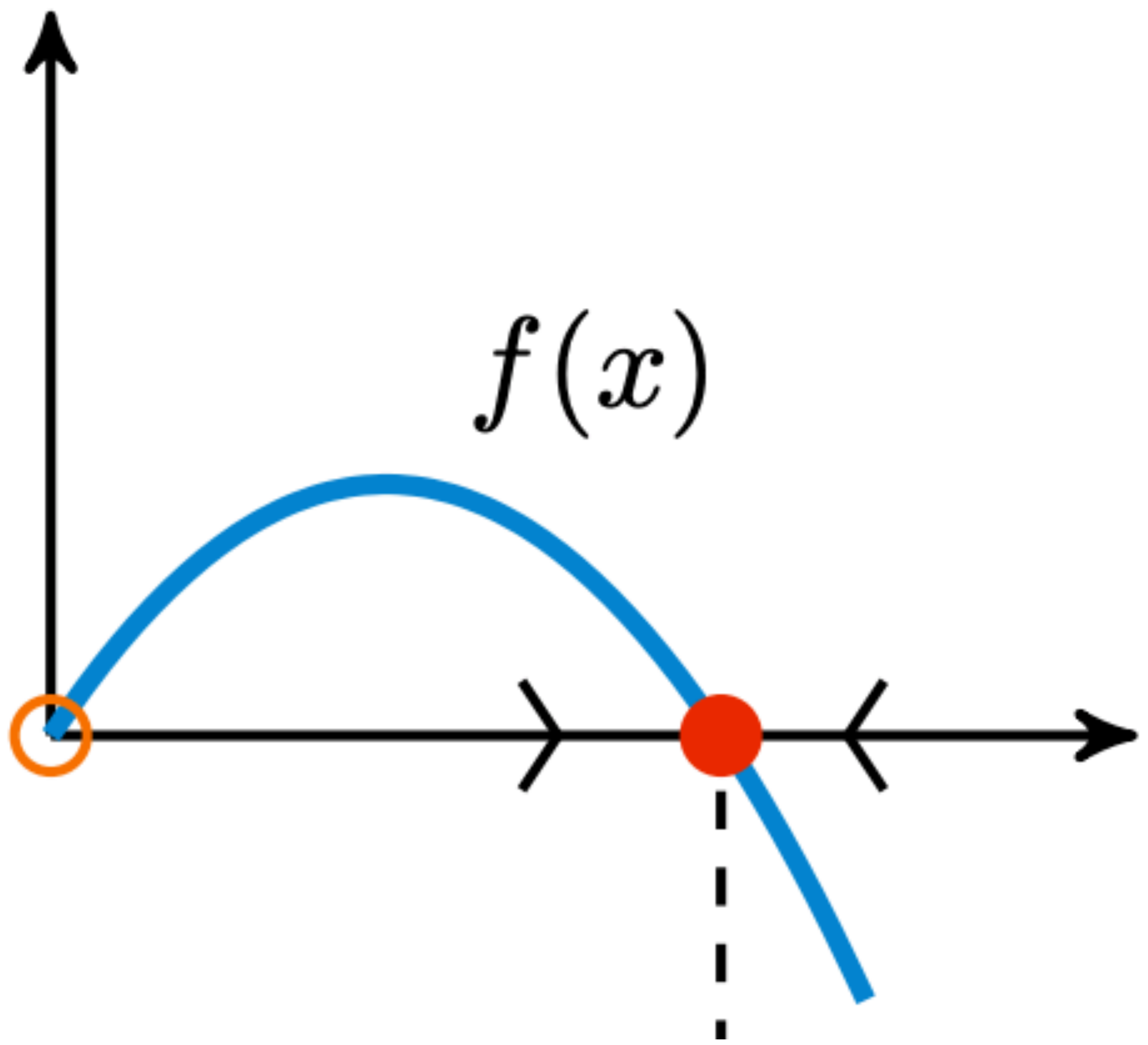
Examples

1. Linear System

2. Nonlinear system (Logistic Growth Model)



Linearization



Continuous Time Linear Systems

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

Stable Equilibrium

Discrete Time Linear Systems

$$\left.\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C_k x_k + D_k u_k \end{aligned}\right\}$$

DT state transition matrix: $\Phi(k, k_0) = A_{k-1} A_{k-2} \cdots A_{k_0}$

$\forall\ k_0,\ \Phi(k+1, k_0) = A_k \Phi(k, k_0),\ k = k_0, k_0+1, \ldots,\ \Phi(k_0, k_0) = I$

Solution:

$$x_k = \Phi(k, k_0) x_0 + \sum_{\ell=k_0}^{k-1} \Phi(k, \ell+1) B_\ell u_\ell$$

$$y_k = C_k \Phi(k, k_0) x_0 + C_k \left(\sum_{\ell=k_0}^{k-1} \Phi(k, \ell+1) B_\ell u_\ell \right) + D_k u_k$$