Mod1-RL2: Solutions to LTV Systems & Fundamental Theorem of ODEs

References:

- Chapter 3 Callier & Desoer **[C&D]**
- Chapter 5 (LTV), Chapter 6 (LTI) Hespanha **[JH]**

Linear Time Varying (LTV) **Autonomous** DE

$$\dot{x} = f(x, t)$$

Let $\mathscr{D} \subseteq \mathbb{R}_+$ be a set which contains at most a finite number of points per unit interval.

The function $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ satisfies two assumptions:

- (**A1**) For each $x \in \mathbb{R}^n$, the map $t \in \mathbb{R}_+ \backslash \mathscr{D} \mapsto f(x, t) \in \mathbb{R}^n$ is continuous and for any $\tau \in \mathscr{D}$, the following limits exists.

$$\lim_{\tau \to \tau_-} f(x, \tau) := f(x, \tau_-) \in \mathbb{R}^n \quad \text{and} \quad \lim_{\tau \to \tau_+} f(x, \tau) := f(x, \tau_+) \in \mathbb{R}^n$$

- (**A2**) For any $t \in \mathbb{R}_+$, the function $f(\,\cdot\,, t)$ is Lipschitz continuous – i.e., $\exists$ a piecewise continuous function $g : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|f(\xi, t) - f(\xi', t)\| \leq g(t)\|\xi - \xi'\| \quad \forall t \in \mathbb{R}_+ \quad \forall \xi, \xi' \in \mathbb{R}^n$$

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Fundamental Theorem of ODEs (FTO): Existence and Uniqueness

Applying FTO to LTV **Systems**

Existence of the State Transition Map (Flow)

Zero State and Zero Input Maps

Recall [510]: Matrix Representation Theorem: Any linear transformation $\mathscr{T} : U \to V$ on vector spaces can be represented as a matrix

State Transition Matrix

State Transition Matrix: Example LTI Systems