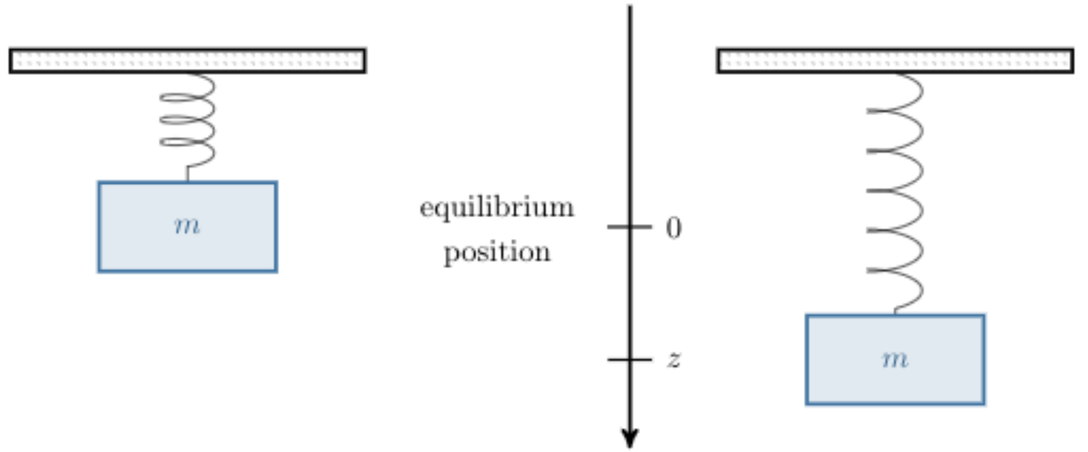
Mod1-RL1: Solutions to ODEs

Solutions to scalar linear differential equations

- The simplest ordinary differential equation (ODE) is a scalar time-invariant linear homogeneous differential equation

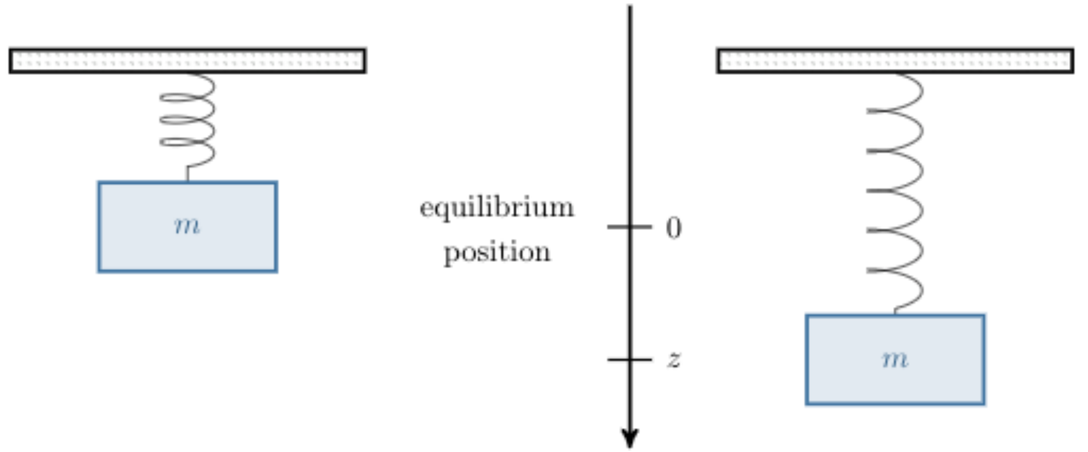
Generalize to higher dimensions

- Consider a second order mechanical system
- Hooke's Law says that if the spring is stretched or compressed z units from its natural length, then it exerts a force proportional to z—namely,



Generalize to higher dimensions

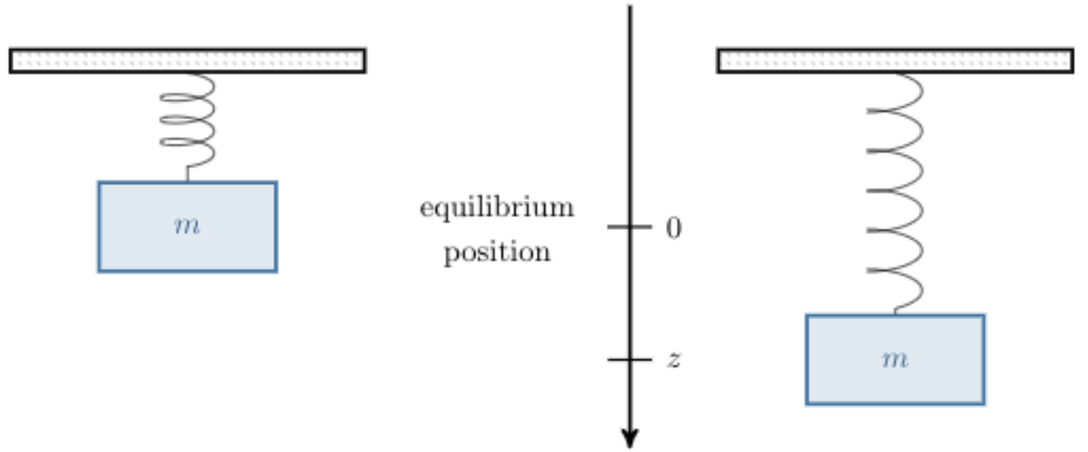
- State-space model



Aside: Matrix Exponential

Generalize to higher dimensions

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



- Eigenpairs of A:

State-Space: General LTI System

Solution to LTI System

Discrete Time LTI

Solution