EE445 Final review/practice

References:

- [VMLS]: Chapters 1-15 (except 9)
- [OM, Calafiore & El-Ghaoui]: See Module 3 refs, and
  Chapter 8, sections 8.1-8.3 (except 8.2.3)
  Chapter 13 (sections 13.1, 13.2, 13.3.1-5)

HW6, Prob 2

Weighted least-squares cost as a function of weights. Let $a_1, \ldots, a_n \in \mathbf{R}^m$. In weighted LS, we minimize the objective $\sum_{i=1}^n w_i(a_i^T x - b_i)^2$ over $x \in \mathbf{R}^m$. Define the *optimal weighted least squares cost* as

$$g(w) = \min_x \sum_{i=1}^n w_i(a_i^T x - b_i)^2,$$

with $\mathbf{dom}\, g = \{w \mid g(w) > -\infty\}$. Show that $g(w)$ is concave in w.

HW6, Prob 3

Some measure of 'spread' of entries in a vector $x \in \mathbf{R}^n$:

1. $\phi_{\text{sprd}}(x) = \max_{i=1,\ldots,n} x_i - \min_{i=1,\ldots,n} x_i$

2. *standard deviation*, defined (recall Module 1, Lec. 2) as

$$\phi_{\text{stdev}}(x) = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n} \sum_{i=1}^{n} x_i \right)^2 \right)^{1/2}.$$

3. *average absolute deviation from the median* of the values:

$$\phi_{\text{aamd}}(x) = (1/n) \sum_{i=1}^{n} |x_i - \text{med}(x)|.$$

More convex sets

1. Is the set $\{x \in \mathbf{R}^n \mid a \leq \|x - x_0\|_2 \leq b\}$ with $b > 0$, convex when
   - $a > 0$?
   - $a = 0$?

   Draw a (2D) picture.

2. Is the set $\{(x, t) \in \mathbf{R}^{n+1} \mid \|x - x_0\|_2 \leq t, \text{ for all } a \leq t \leq b\}$ convex for $a > 0$? Draw a (3D) picture.