

All hw should be uploaded to canvas as a *pdf*. Make sure that if you scan your handwritten notes that they are legible and appropriately oriented. If you use an online resource to solve any problem, please appropriately cite that source.

Solve these following problems from the [VMLS](#) book ([linked here](#)):

Problem 1. (VMLS 8.8, Interpolation of rational functions.)

Problem 2. (VMLS 8.16, Bilinear interpolation.)

Problem 3. (VMLS 10.13, Laplacian matrix of a graph.)

Problem 4. (VMLS 10.16, Covariance matrix.)

Problem 5. (VMLS 11.12, Combinations of invertible matrices.)

Problem 6. (VMLS 11.24, Simultaneous left-inverse.)

Problem 7. (Number of paths of a given length in a directed graph.) Consider a directed graph with n nodes, specified by its $n \times n$ adjacency matrix A (see VMLS, page 112). What is the total number of paths of length 10 in the graph? (See VMLS, section 10.3). Give an expression for this number that involves the matrix A and standard matrix operations (like matrix-matrix or matrix-vector multiplication, powers, transpose). Your solution should use only matrix and vector notation, and not involve a sum.

Problem 8. (Auto-Regressive Time Series Model.) In Lecture 3, we saw an example of one type of time series model, an input-output convolution system. Here we explore the data structure in another common time-series model.

Suppose we have T noiseless data samples z_1, \dots, z_T from an auto-regressive model with memory M (see [VMLS](#), page 164 under Markov model). Find the matrix A and vector y for which $A\beta = y$. Show that A is a Toeplitz matrix, i.e., entries A_{ij} with the same value of $i - j$ are the same.